UNIT-I

DIFFERENTIAL EQUATIONS OF FIRST ORDER AND THEIR APPLICATIONS
# UNIT INDEX

## UNIT-I

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Lecture-1

INTRODUCTION

- An equation involving a dependent variable and its derivatives with respect to one or more independent variables is called a Differential Equation.

- Example 1: $y'' + 2y = 0$

- Example 2: $y_2 - 2y_1 + y = 23$

- Example 3:
  \[
  \frac{d^2 y}{dx^2} + \frac{dy}{dx} - y = 1
  \]
TYPES OF DIFFERENTIAL EQUATION

- ORDINARY DIFFERENTIAL EQUATION:
A differential equation is said to be ordinary, if the derivatives in the equation are ordinary derivatives.

Example:
\[ \frac{d^2 y}{dx^2} - \frac{dy}{dx} + y = 1 \]

- PARTIAL DIFFERENTIAL EQUATION:
A differential equation is said to be partial if the derivatives in the equation have reference to two or more independent variables.

Example:
\[ \frac{\partial^4 y}{\partial x^4} + \frac{\partial y}{\partial x} + y = 1 \]
LECTURE-2

DEFINITIONS

• ORDER OF A DIFFERENTIAL EQUATION:
A differential equation is said to be of order $n$, if the $n^{th}$ derivative is the highest derivative in that equation.

  Example: Order of $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 2$ is 2

• DEGREE OF A DIFFERENTIAL EQUATION:
If the given differential equation is a polynomial in $y^{(n)}$, then the highest degree of $y^{(n)}$ is defined as the degree of the differential equation.

  Example: Degree of $\left(\frac{dy}{dx}\right)^4 + y = 3$ is 4
SOLUTION OF A DIFFERENTIAL EQUATION

- SOLUTION: Any relation connecting the variables of an equation and not involving their derivatives, which satisfies the given differential equation is called a solution.
- GENERAL SOLUTION: A solution of a differential equation in which the number of arbitrary constant is equal to the order of the equation is called a general or complete solution or complete primitive of the equation.
- Example: $y = Ax + B$
- PARTICULAR SOLUTION: The solution obtained by giving particular values to the arbitrary constants of the general solution, is called a particular solution of the equation.
- Example: $y = 3x + 5$
Lecture-3

EXACT DIFFERENTIAL EQUATION

- Let $M(x,y)dx + N(x,y)dy = 0$ be a first order and first degree differential equation where $M$ and $N$ are real valued functions for some $x, y$. Then the equation $Mdx + Ndy = 0$ is said to be an exact differential equation if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

- Example: $(2y \sin x + \cos y)dx = (x \sin y + 2\cos x + \tan y)dy$
Lecture-4

Working rule to solve an exact equation

- **STEP 1:** Check the condition for exactness, \( \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \), if exact proceed to step 2.
- **STEP 2:** After checking that the equation is exact, solution can be obtained as

\[
\int Mdx + \int (N \text{ terms not containing } x)dy = C
\]

(y is constant)
Lecture-5

INTEGRATING FACTOR

- Let \( Mdx + Ndy = 0 \) be not an exact differential equation. Then \( Mdx + Ndy = 0 \) can be made exact by multiplying it with a suitable function called an integrating factor.

- **Example 1:** \( ydx - xdy = 0 \) is not an exact equation. Here \( 1/x^2 \) is an integrating factor.

- **Example 2:** \( y(x^2y^2 + 2)dx + x(2 - 2x^2y^2)dy = 0 \) is not an exact equation. Here \( 1/(3x^3y^3) \) is an integrating factor.
METHODS TO FIND INTEGRATING FACTORS

- **METHOD 1**: With some experience integrating factors can be found by inspection. That is, we have to use some known differential formulae.

**Example 1**: \( d(xy) = xdy + ydx \)

**Example 2**: \( d(x/y) = (ydx - xdy)/y^2 \)

**Example 3**: \( d[\log(x^2 + y^2)] = 2(xdx + ydy)/(x^2 + y^2) \)
METHODS TO FIND INTEGRATING FACTORS

METHOD 2: If $Mdx + Ndy = 0$ is a non-exact but homogeneous differential equation and $Mx + Ny \neq 0$ then $1/(Mx + Ny)$ is an integrating factor of $Mdx + Ndy = 0$.

Example 1: $x^2 ydx - (x^3 + y^3)dy = 0$ is a non-exact homogeneous equation. Here I.F. = $-1/y^4$

Example 2: $y^2dx + (x^2 - xy - y^2)dy = 0$ is a non-exact homogeneous equation. Here I.F. = $1/(x^2 y - y^3)$
METHODS TO FIND INTEGRATING FACTORS

- **METHOD 3:** If the equation \( Mdx + Ndy = 0 \) is of the form \( y.f(xy) \, dx + x.g(xy) \, dy = 0 \) and \( Mx - Ny \neq 0 \) then \( \frac{1}{Mx - Ny} \) is an integrating factor of \( Mdx + Ndy = 0 \).

**Example 1:** \( y(x^2y^2+2)dx + x(2-2x^2 \, y^2)dy = 0 \) is non-exact and in the above form. Here I.F=\( \frac{1}{3x^3 \, y^3} \)

**Example 2:** \( (xysinxy+cosxy)ydx+(xysinxy-cosxy)x dy = 0 \) is non-exact and in the above form. Here I.F=\( \frac{1}{2xycosxy} \)
Lecture-9

METHODS TO FIND INTEGRATING FACTORS

• METHOD 4: If there exists a continuous single
variable function $f(x)$ such that \[
\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = f(x)
\]
then $e^{\int f(x)dx}$ is an integrating factor of

$Mdx + Ndy = 0$

• Example 1: $2xydy - (x^2 + y^2 + 1)dx = 0$ is non-exact and

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{-2}{x}$ . Here I.F=$1/x^2$

• Example 2: $(3xy - 2ay^2)dx + (x^2 - 2axy) = 0$ is non-exact

and $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{1}{x}$ . Here I.F=$x$
Lecture-10
METHODS TO FIND INTEGRATING FACTORS

• METHOD 5: If there exists a continuous single variable function \( f(y) \) such that \( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = g(y) \), then \( e^{\int g(y) dy} \) is an integrating factor of \( Mdx + Ndy = 0 \).

Example 1: \((xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0\) is a non-exact equation and \( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = \frac{1}{y} \). Here I.F = y

Example 2: \((y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0\) is a non-exact equation and \( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = -\frac{3}{y} \). Here I.F = \( \frac{1}{y^3} \)
Lecture-11

LINEAR EQUATION

- An equation of the form $y' + Py = Q$ is called a linear differential equation, where $P$ and $Q$ are constants or functions of $x$

- Integrating Factor (I.F.) = $e^{\int P \, dx}$

- Solution is $y(I.F) = Q(I.F) \, dx + C$

Example 1: $x \frac{dy}{dx} + y = \log x$. Here $I.F=x$ and solution is $xy=x(\log x-1)+C$

Example 2: $\frac{dy}{dx} + 2xy = e^{-x}$. Here $I.F=e^x$ and solution is $ye^x = x + C$
BERNOULLI’S LINEAR EQUATION

An equation of the form $y' + Py = Qy^n$ is called a Bernoulli’s linear differential equation. This differential equation can be solved by reducing it to the linear differential equation. For this dividing above equation by $y^n$

**Example 1:** $x \frac{dy}{dx} + y = x^2 y^6$. Here I.F=$1/x^5$ and solution is $1/(xy)^5 = 5x^3/2 + Cx^5$

**Example 2:** $\frac{dy}{dx} + \frac{y}{x} = y^2 x \sin x$. Here I.F=$1/x$ and solution is $1/xy = \cos x + C$
Lecture-13
ORTHOGONAL TRAJECTORIES

- If two families of curves are such that each member of family cuts each member of the other family at right angles, then the members of one family are known as the orthogonal trajectories of the other family.

- **Example 1:** The orthogonal trajectory of the family of parabolas through origin and foci on y-axis is $\frac{x^2}{2c} + \frac{y^2}{c} = 1$

- **Example 2:** The orthogonal trajectory of rectangular hyperbolas is $xy = c^2$
PROCEDURE TO FIND ORTHOGONAL TRAJECTORIES

- Suppose $f(x, y, c) = 0$ is the given family of curves, where $c$ is the constant.
- **STEP 1:** Form the differential equation by eliminating the arbitrary constant.
- **STEP 2:** Replace $y'$ by $-1/y'$ in the above equation.
- **STEP 3:** Solve the above differential equation.
NEWTON’S LAW OF COOLING

The rate at which the temperature of a hot body decreases is proportional to the difference between the temperature of the body and the temperature of the surrounding air.

\[ \theta' \propto (\theta - \theta_0) \]

**Example:** If a body is originally at 80°C and cools down to 60°C in 20 min. If the temperature of the air is at 40°C then the temperature of the body after 40 min is 50°C
Lecture-15
LAW OF NATURAL GROWTH

- When a natural substance increases in Magnitude as a result of some action which affects all parts equally, the rate of increase depends on the amount of the substance present.

\[ N' = kN \]

- Example: If the number \( N \) of bacteria in a culture grew at a rate proportional to \( N \). The value of \( N \) was initially 100 and increased to 332 in 1 hour. Then the value of \( N \) after one and half hour is 605
LAW OF NATURAL DECAY

• The rate of decrease or decay of any substance is proportion to N the number present at time. 
  \[ N' = -k N \]

• Example: A radioactive substance disintegrates at a rate proportional to its mass. When mass is 10gms, the rate of disintegration is 0.051gms per day. The mass is reduced to 10 to 5gms in 136 days.
UNIT – II

LINEAR DIFFERENTIAL EQUATIONS OF SECOND AND HIGHER ORDER AND THEIR APPLICATIONS
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INTRODUCTION

• An equation of the form
  \[ D^n y + k_1 D^{n-1} y + \ldots + k_n y = X \]
  Where \( k_1, \ldots, k_n \) are real constants and \( X \) is a continuous function of \( x \) is called an ordinary linear equation of order \( n \) with constant coefficients.
  Its complete solution is
  \[ y = C.F + P.I \]
  where C.F is a Complementary Function and P.I is a Particular Integral.

• Example: \( \frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + 4y = \sin x \)
COMPLEMENTARY FUNCTION

- If roots are real and distinct then
  \[ C.F = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \ldots + c_k e^{m_k x} \]

- Example 1: If roots of an auxiliary equation are 1, 2, 3 then
  \[ C.F = c_1 e^{x} + c_2 e^{2x} + c_3 e^{3x} \]

- Example 2: For a differential equation \((D-1)(D+1)y=0\), roots are -1 and 1. Hence
  \[ C.F = c_1 e^{-x} + c_2 e^{x} \]
Lecture-2

COMPLEMENTARY FUNCTION

• If roots are real and equal then
  \[ C.F = \left( c_1 + c_2 x + c_3 x^2 + \ldots + c_k x^{k-1} \right) e^{mx} \]

Example 1: The roots of a differential equation \((D-1)^3 y=0\) are 1,1,1. Hence C.F. = \((c_1 + c_2 x + c_3 x^2)e^x\)

• Example 2: The roots of a differential equation \((D+1)^2 y=0\) are -1,-1. Hence C.F. = \((c_1 + c_2 x)e^{-x}\)
Lecture-3
COMPLEMENTARY FUNCTION

- If two roots are real and equal and rest are real and different then
  \[ C.F. = (c_1 + c_2 x) e^{m_1 x} + c_3 x e^{m_3 x} + \ldots \]

- Example: The roots of a differential equation \((D-2)^2(D+1)y=0\) are 2,2,-1. Hence
  \[ C.F. = (c_1 + c_2 x)e^{2x} + c_3 e^{-x} \]
Lecture-4

COMPLEMENTARY FUNCTION

- If roots of Auxiliary equation are complex say p+iq and p-iq then
  C.F = e^{px}(c_1 \cos qx + c_2 \sin qx)

- Example: The roots of a differential equation (D^2+1)y=0 are o+i(1) and o-i(1). Hence
  C.F = e^{ox}(c_1 \cos x + c_2 \sin x) = (c_1 \cos x + c_2 \sin x)
Lecture-5

COMPLEMENTARY FUNCTION

- A pair of conjugate complex roots say $p+iq$ and $p-iq$ are repeated twice then
  \[ C.F = e^{px}((c_1 + c_2 x)\cos qx + (c_3 + c_4 x)\sin qx) \]
- **Example**: The roots of a differential equation $(D^2-D+1)^2 y = 0$ are $(\frac{1}{2})+i(1.7/2)$ and $(\frac{1}{2})-i(1.7/2)$ repeated twice. Hence
  \[ C.F = e^{1/2x}(c_1 + c_2 x) \cos (1.7/2)x + (c_3 + c_4 x) \sin (1.7/2)x \]
Lecture-6

PARTICULAR INTEGRAL

- When \( X = e^{ax} \) put \( D = a \) in Particular Integral.
  - If \( f(a) \neq 0 \) then P.I. will be calculated directly.
  - If \( f(a) = 0 \) then multiply P.I. by \( x \) and differentiate denominator. Again put \( D = a \). Repeat the same process.

- Example 1: \( y + 5y' + 6y = e^x \). Here P.I. = \( e^x/12 \)
- Example 2: \( 4D^2y + 4Dy - 3y = e^{2x} \). Here P.I. = \( e^{2x}/21 \)
Lecture-7
PARTICULAR INTEGRAL

- When \( X = \sin(ax) \) or \( \cos(ax) \) or \( \sin(ax+b) \) or \( \cos(ax+b) \) then put \( D^2 = -a^2 \) in Particular Integral.

- Example 1: \( D^2y - 3Dy + 2y = \cos3x \). Here
  \[ P.I = \frac{(9\sin3x + 7\cos3x)}{130} \]

- Example 2: \( (D^2 + D + 1)y = \sin2x \). Here
  \[ P.I = \frac{-(2\cos2x + 3\sin2x)}{13} \]
Lecture-8

PARTICULAR INTEGRAL

- When \( X = x^k \) or in the form of polynomial then convert \( f(D) \) into the form of binomial expansion from which we can obtain Particular Integral.

- Example 1: \((D^2+D+1)y=x^3\). Here P.I=\(x^3 - 3x^2 + 6\)

- Example 2: \((D^2+D)y=x^2+2x+4\). Here P.I=(\(x^3/3\))+4x
PARTICULAR INTEGRAL

- When $X = e^{ax}v$ then put $D = D + a$ and take out $e^{ax}$ to the left of $f(D)$. Now using previous methods we can obtain Particular Integral.

- Example 1: $(D^4 - 1)y = e^x \cos x$. Here $P.I = -e^x \cos x / 5$

- Example 2: $(D^2 - 3D + 2)y = xe^{3x} + \sin 2x$. Here $P.I = \frac{e^{3x}}{2} \left( x - \frac{3}{2} \right) + \frac{1}{20} (3 \cos 2x - \sin 2x)$
Lecture-10

PARTICULAR INTEGRAL

- When $X = x \cdot v$ then
  \[ \text{P.I.} = \left\{ x - \frac{f''(D)}{f(D)} \right\} v \]

- Example 1: \((D^2 + 2D + 1)y = x \cos x. \ \text{Here} \]
  \[ \text{P.I.} = \frac{x \sin x}{2} + \frac{(\cos x - \sin x)}{2} \]

- Example 2: \((D^2 + 3D + 2)y = x e^x \sin x. \ \text{Here} \]
  \[ \text{P.I.} = e^x \left[ \frac{x(\sin x - \cos x)}{10} - \frac{\sin x}{25} + \frac{\cos x}{10} \right] \]
Lecture-11

PARTICULAR INTEGRAL

- When *X* is any other function then Particular Integral can be obtained by resolving $1/f(D)$ into partial fractions.

- **Example 1**: $(D^2 + a^2)y = \sec ax$. Here
  
  \[ P.I = x \sin ax/a + \cos ax \log(\cos ax)/a^2 \]
Lecture-12
CAUCHY’S LINEAR EQUATION

- Its general form is \( x^n D^n y + \ldots + y = X \) then to solve this equation put \( x = e^z \) and convert into ordinary form.

- Example 1: \( x^2 D^2 y + xDy + y = 1 \)

- Example 2: \( x^3 D^3 y + 3x^2 D^2 y + 2xDy + 6y = x^2 \)
LEGENDRE’S LINEAR EQUATION

- Its general form is \((ax + b)^n D^n y + \ldots + y = X\)
  - Then to solve this equation put \(ax + b = e^z\) and convert into ordinary form.
- **Example 1:** \((x+1)^2 D^2 y - 3(x+1)Dy + 4y = x^2 + x + 1\)
- **Example 2:** \((2x-1)^3 D^3 y + (2x-1)Dy - 2y = x\)
Lecture-14  
METHOD OF VARIATION OF PARAMETERS  
• Its general form is $D^2 y + P Dy + Q = R$  
where P, Q, R are real valued functions of $x$.  
Let $C.F = C_1 u + C_2 v$  
P.I = $Au + Bv$  
• Example 1: $(D^2 +1)y=\text{Cosec}x$. Here $A=-x$, $B=\log(\text{Sin}x)$  
• Example 2: $(D^2 +1)y=\text{Cos}x$. Here $A=\text{Cos}2x/4$, $B=(x+\text{Sin}2x)/2$
Simple Harmonic Motion

- Any vibrating system where the restoring force is proportional to the negative of the displacement is in simple harmonic motion (SHM), and is often called a simple harmonic oscillator (SHO).
Substituting $F = kx$ into Newton’s second law gives the equation of motion:

$$F = kx = m \frac{dv}{dt} = m \frac{d}{dt} \frac{dx}{dt} = m \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m} x = 0$$

with solutions of the form:

$$x = A \cos(\omega t + \phi).$$
The constants $A$ and $\phi$ will be determined by initial conditions; $A$ is the amplitude, and $\phi$ gives the phase of the motion at $t = 0$.

The velocity and acceleration can be found by differentiating the displacement:

\[ v = \frac{dx}{dt} = \frac{d}{dt} [A \cos(\omega t + \phi)] = -\omega A \sin(\omega t + \phi). \]

\[ a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi). \]
UNIT – III

Functions of Single Variable and their applications and Multiple Integrals
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Here we study about Mean value theorems.

Continuous function: If limit of $f(x)$ as $x$ tends $c$ is $f(c)$ then the function $f(x)$ is known as continuous function. Otherwise the function is known as discontinuous function.

Example: If $f(x)$ is a polynomial function then it is continuous.
Lecture-2

ROLLE’S MEAN VALUE THEOREM

Let $f(x)$ be a function such that
1) it is continuous in closed interval $[a, b]$
2) it is differentiable in open interval $(a, b)$ and
3) $f(a)=f(b)$

Then there exists at least one point $c$ in open interval $(a, b)$ such that $f'(c)=0$

Example: $f(x)=(x+2)^3(x-3)^4$ in $[-2,3]$. Here $c=-2$ or $3$ or $1/7$ where $c=1/7$ is in $(-2,3)$
Lecture-3
LAGRANGE’S MEAN VALUE THEOREM

- Let \( f(x) \) be a function such that
  1) it is continuous in closed interval \([a, b]\) and
  2) it is differentiable in open interval \((a, b)\)

Then there exists at least one point \( c \) in open interval \((a, b)\) such that

\[
f'(c) = \frac{f(b) - f(a)}{b - a}\]

Example: \( f(x) = x^3 - x^2 - 5x + 3 \) in \([0, 4]\). Here
\( c = 1 + \sqrt{37}/3 \in (0, 4) \)
Lecture-4

CAUCHY’S MEAN VALUE THEOREM

If \( f : [a, b] \rightarrow \mathbb{R}, g : [a, b] \rightarrow \mathbb{R} \) are such that

1) \( f, g \) are continuous on \([a, b]\)
2) \( f, g \) are differentiable on \((a, b)\) and
3) \( g'(x) \neq 0 \) for all \( x \in (a, b) \) then there exists at least one point \( c \) in \((a, b)\) such that

\[
\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}
\]

Example: \( f(x) = \sqrt{x}, g(x) = 1/\sqrt{x} \) in \([a, b]\). Here \( c = \sqrt{ab} \in (a, b) \)
Lecture-5

TAYLOR’S THEOREM

- If \( f : [a, b] \rightarrow \mathbb{R} \) is such that
  1) \( f^{(n-1)} \) is continuous on \([a, b]\)
  2) \( f^{(n-1)} \) is derivable on \((a, b)\) then there exists a point \( c \in (a, b) \) such that
  \[
  f(b) = f(a) + \frac{f'(a)}{1!}(b-a) + \frac{f''(a)}{2!}(b-a)^2 + \ldots.
  \]

- Example: \( f(x) = e^x \). Here Taylor’s expansion at \( x=0 \) is
  \[
  1 + x + \frac{x^2}{2!} + \ldots.
  \]
Lecture-6
MACLAURIN’S THEOREM

If \( f: [0, x] \rightarrow \mathbb{R} \) is such that
1) \( f^{(n-1)} \) is continuous on \([0,x]\)
2) \( f^{(n-1)} \) is derivable on \((0,x)\) then there exists a real number \((0,1)\) such that

\[ f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \ldots \]

**Example:** \( f(x) = \cos x \). Here Maclaurin’s expansion is

\[ 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots \]
Lecture-7
FUNCTIONS OF SEVERAL VARIABLES

- We have already studied the notion of limit, continuity and differentiation in relation of functions of a single variable. In this chapter we introduce the notion of a function of several variables i.e., function of two or more variables.

- \textit{Example 1: Area } A = ab
- \textit{Example 2: Volume } V = abh
Lecture-8

DEFINITIONS

• Neighbourhood of a point \((a,b)\): A set of points lying within a circle of radius \(r\) centered at \((ab)\) is called a neighbourhood of \((a,b)\) surrounded by the circular region.

• Limit of a function: A function \(f(x,y)\) is said to tend to the limit \(l\) as \((x,y)\) tends to \((a,b)\) if corresponding to any given positive number \(p\) there exists a positive number \(q\) such that \(f(x,y) - l < p\) for all points \((x,y)\) whenever \(x - a \leq q\) and \(y - b \leq q\)
Lecture-9

**JACOBIAN**

- Let \( u = u(x,y) \), \( v = v(x,y) \). Then these two simultaneous relations constitute a transformation from \((x, y)\) to \((u, v)\). Jacobian of \( u, v \) w.r.t \( x, y \) is denoted by \( \frac{\partial (u, v)}{\partial (x, y)} \) or \( \frac{\partial (u, v)}{\partial (x, y)} \)

- **Example**: \( x = r \cos \theta, y = r \sin \theta \) then \( \frac{\partial (x, y)}{\partial (r, \theta)} = r \)

  and \( \frac{\partial (r, \theta)}{\partial (x, y)} = \frac{1}{r} \)
Lecture-10
MAXIMUM AND MINIMUM OF FUNCTIONS OF TWO VARIABLES

Let $f(x,y)$ be a function of two variables $x$ and $y$. At $x=a$, $y=b$, $f(x,y)$ is said to have maximum or minimum value, if $f(a,b)>f(a+h,b+k)$ or $f(a,b)<f(a+h,b+k)$ respectively where $h$ and $k$ are small values.

Example: The maximum value of $f(x,y)=x^3 +3xy^2 -3y^2+4$ is $36$ and minimum value is $-36$
Lecture-11

EXTREME VALUE

- $f(a,b)$ is said to be an extreme value of $f$ if it is a maximum or minimum value.

- **Example 1:** The extreme values of $u=x^2y^2 - 5x^2 - 8xy - 5y^2$ are -8 and -80

- **Example 2:** The extreme value of $x^2 + y^2 + 6x + 12$ is 3
Lecture-12
LAGRANGE’S METHOD OF UNDETERMINED MULTIPLIERS

Suppose it is required to find the extremum for the function $f(x,y,z)=0$ subject to the condition $\phi(x,y,z)=0$

1) Form Lagrangian function $F=f+\lambda \phi$
2) Obtain $F_x=0, F_y=0, F_z=0$
3) Solve the above 3 equations along with condition.

Example: The minimum value of $x^2 + y^2 + z^2$ with $xyz=a^3$ is $3a^2$
Lecture-13
CURVATURE

- Curvature is a concept introduced to quantify the bending of curves at any point.
- **Note**: The curvature at any point of the circle is equal to the reciprocal of its radius. The curvature of the circle decreases as the radius increases.
- **Theorem**: The curvature of a circle at any point on it is a constant.
Lecture-14

RADIUS OF CURVATURE

- The reciprocal of the curvature at any point of a curve is defined to be the radius of curvature at that point.

- Note: The radius of curvature of a circle of radius \(r\) at any point is \(r\).

- Example 1: The radius of curvature at any point on the curve \(xy = c^2\) is \((x^2 + y^2)^{3/2}/2xy\)

- Example 2: The radius of curvature at \((3a/2, 3a/2)\) of the curve \(x^3 + y^3 = 3axy\) is \(3\sqrt{2a}/16\)
Lecture-15
Formulae for RADIUS OF CURVATURE

- In cartesian form, \( \rho = \frac{(1+(y')^2)^{3/2}}{y''} \)
- In polar form, \( \rho = \frac{(r^2 + r_1^2)^{3/2}}{(r^2 + 2r_1^2 - rr_2)} \)

Example 1: \( r = a(1 - \cos \theta) \). Here \( \theta = \frac{4\sin \theta}{3a} \)

Example 2: \( x = a(\theta + \sin \theta), y = a(1 - \cos \theta) \) at \( \pi / 2 \).
Here, \( \rho = 2\sqrt{2a} \)

Example 3: \( x = a(\cos \theta + t \sin \theta), y = a(\sin \theta - t \cos \theta) \).
Here \( \rho = a \theta \)
Lecture-16
CENTRE OF CURVATURE

- The centre of curvature at any point \( P \) on a curve is the point which lies on the positive direction of the normal at \( P \) and is at a distance from it. The centre of curvature at any point of a curve lies on the side towards which side the curve is concave.

- Example: Centre of curvature at \( (a/4, a/4) \) of the curve \( \sqrt{x} + \sqrt{y} = \sqrt{a} \) is \( a^2/2 \)
Lecture-17

Formula for CENTRE OF CURVATURE

\[ X = x - \frac{y_1(1+y_1^2)}{y_2}, \quad Y = y + \frac{1+y_1^2}{Y_2} \]

Example 1: \( x^3 + y^3 = 2 \) at (1,1). Here \( X=1/2, \ Y=1/2 \)

Example 2: \( x=a(\theta - \sin \theta), y=a(1-\cos \theta) \). Here \( X=a(\theta + \sin \theta), \ Y=-a(1-\cos \theta) \)
Lecture-18
CIRCLE OF CURVATURE

The circle of curvature at any point of a curve is the circle with centre at the centre of curvature at P and radius equal to the radius of curvature at the point. If (X,Y) be the centre and \( \rho \) be the radius of curvature, then the equation of the circle of curvature at the given point (x, y) is given by \((x-X)^2 + (y-Y)^2 = \rho^2\)

- Example: \( y=x^3+2x^2+x+1 \) at (0,1). Here the circle of curvature is \( x^2 + y^2 + x - 3y + 2 = 0 \)
Lecture-19

E VOLU T E

- The locus of the centre of curvature C of a variable point P on a curve is called the evolute of the curve. The curve itself is called Involute of the evolute.

Example 1: \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \). Here the evolute is \( (ax)^{2/3} - (by)^{2/3} = (a^2 + b^2)^{2/3} \)

Example 2: \( x = a \cos \theta \), \( y = b \sin \theta \). Here the evolute is \( (ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3} \)
Lecture-20

ENVELOPE

Let \( f(x,y,c) \) be a function of three variables \( x,y,c \). A curve which touches each member of a given family of curves is called envelope of that family.

Example 1: \( y=mx+a/m \) where \( m \) is parameter. Here envelope is \( y^2 = 4ax \)

Example 2: \( (x/a) \cos \theta + (y/b) \sin \theta = 1 \) where \( \theta \) is parameter and \( a,b \) are constants. Here envelope is \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \)
Lecture-21
MULTIPLE INTEGRALS

• Let $y = f(x)$ be a function of one variable defined and bounded on $[a, b]$. Let $[a, b]$ be divided into $n$ subintervals by points $x_0, \ldots, x_n$ such that $a = x_0, \ldots, x_n = b$. The generalization of this definition to two dimensions is called a double integral and to three dimensions is called a triple integral.
Lecture-22

DOUBLE INTEGRALS

- Double integrals over a region R may be evaluated by two successive integrations. Suppose the region R cannot be represented by those inequalities, and the region R can be subdivided into finitely many portions which have that property, we may integrate $f(x,y)$ over each portion separately and add the results. This will give the value of the double integral.
Lecture-23

CHANGE OF VARIABLES IN DOUBLE INTEGRAL

- Sometimes the evaluation of a double or triple integral with its present form may not be simple to evaluate. By choice of an appropriate coordinate system, a given integral can be transformed into a simpler integral involving the new variables. In this case we assume that $x=r \cos \theta$, $y=r \sin \theta$ and $dxdy=rdrd\theta$
Lecture-24

CHANGE OF ORDER OF INTEGRATION

Here change of order of integration implies that the change of limits of integration. If the region of integration consists of a vertical strip and slide along $x$-axis then in the changed order a horizontal strip and slide along $y$-axis then in the changed order a horizontal strip and slide along $y$-axis are to be considered and vice-versa. Sometimes we may have to split the region of integration and express the given integral as sum of the integrals over these sub-regions. Sometimes as commented above, the evaluation gets simplified due to the change of order of integration. Always it is better to draw a rough sketch of region of integration.
Lecture-25
TRIPLE INTEGRALS
• The triple integral is evaluated as the repeated integral where the limits of z are \( z_1 , z_2 \) which are either constants or functions of x and y; the y limits \( y_1 , y_2 \) are either constants or functions of x; the x limits \( x_1 , x_2 \) are constants. First \( f(x,y,z) \) is integrated w.r.t. z between z limits keeping x and y are fixed. The resulting expression is integrated w.r.t. y between y limits keeping x constant. The result is finally integrated w.r.t. x from \( x_1 \) to \( x_2 \).
UNIT-IV

LAPLACE TRANSFORMS AND THEIR APPLICATIONS TO ORDINARY DIFFERENTIAL EQUATIONS
## UNIT INDEX
### UNIT-IV

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Lecture-1

DEFINITION

Let $f(t)$ be a function defined for all positive values of $t$. Then the Laplace transform of $f(t)$, denoted by $L\{f(t)\}$ or $f(s)$ is defined by

$$L[f(t)] = \int e^{-st}f(t)dt = F(s)$$

where $s$ is a real or complex number

- Example 1: $L\{1\} = 1/s$
- Example 2: $L\{e^{at}\} = 1/(s-a)$
- Example 3: $L\{\sin at\} = a/(s^2 + a^2)$
FIRST SHIFTING THEOREM

- If \( L\{f(t)\}=F(s) \), then \( L\{e^{at} f(t)\}=f(s-a) \), \( s-a>0 \) is known as a first shifting theorem.

- **Example 1:** By first shifting theorem, the value of \( L\{e^{at} \sin bt\} \) is \( b/[(s-a)^2 +b^2] \)

- **Example 2:** \( L\{e^{at} t^n\}=n!/(s-a)^{n+1} \)

- **Example 3:** \( L\{e^{at} \sinh bt\}=b/[(s-a)^2 -b^2] \)

- **Example 4:** \( L\{e^{-at} \sin bt\}=b/[(s+a)^2 +b^2] \)
Lecture-3
UNIT STEP FUNCTION (HEAVISIDES UNIT FUNCTION)

• The unit step function is defined as

\[
U(t-a) \begin{cases} 
= 0, & \text{if } t < a \\
= 1, & \text{otherwise}
\end{cases}
\]

then \( L\{u(t-a)\} = e^{-as} F(s) \)

• **Example 1:** The laplace transform of 
  \((t-2)^3 \ u(t-2)\) is \(6e^{-2s} / s^4\)

• **Example 2:** The laplace transform of 
  \(e^{-3t} \ u(t-2)\) is \(e^{-2(s+3)} / (s+3)\)
Lecture-4

CHANGE OF SCALE PROPERTY

- If $L\{f(t)\} = F(s)$, then $L\{f(at)\} = \frac{1}{a} F(s/a)$ is known as a change of scale property.

- Example 1: By change of scale property the value of $L\{\sin^2 at\} = \frac{2a^2}{s(s^2 + 4a^2)}$

- Example 2: If $L\{f(t)\} = \frac{1}{se^{-1/s}}$ then by change of scale property the value of $L\{e^{-t} f(3t)\} = \frac{e^{-3/(s+1)}}{(s+1)}$
Lecture-5

LAPLACE TRANSFORM OF INTEGRAL

- If \( L\{f(t)\} = F(s) \) then \( L\{ f(u)du\} = 1/s \) \( f(s) \) is known as Laplace transform of integral.

- **Example 1:** By the integral formula,
  
  \[ L\left[ \int e^{-t}\cos t dt \right] = \frac{(s+1)}{[s^2 + 2s + 2]} \]

- **Example 2:** By the integral formula,
  
  \[ L\left[ \int \int \cosh at dt dt \right] = \frac{1}{[s(s^2 - a^2)]} \]
Lecture-6
LAPLACE TRANSFORM OF $t^n f(t)$

- If $f(t)$ is sectionally continuous and of exponential order and if $L\{f(t)\}=F(s)$ then
  \[ L\{tf(t)\} = -(d/ds)F(s) \]
- In general $L\{t^n f(t)\} = \left(\frac{-1}{s}\right)^n \frac{d^n}{ds^n} F(s)$
- Example 1: By the above formula the value of $L\{t \cos at\}$ is
  \[ \frac{s^2 - a^2}{(s^2 + a^2)^2} \]
- Example 2: By the above formula the value of $L\{te^{-t} \cos ht\}$ is
  \[ \frac{s^2 + 2s + 2}{(s^2 + 2s)^2} \]
Lecture-7

LAPLACE TRANSFORM OF f(t)/t

- If $L\{f(t)\}=F(s)$, then $L\{f(t)/t\} = \int_{s}^{\infty} F(s) ds$, provided the integral exists.

- **Example 1:** By the above formula, the value of $L\{\sin t/t\} = \cot^{-1} s$

- **Example 2:** By the above formula, the value of $L\{(e^{-at} - e^{-bt})/t\} = \log[(s+b)/(s+a)]$
Lecture-8

LAPLACE TRANSFORM OF PERIODIC FUNCTION

- PERIODIC FUNCTION: A function $f(t)$ is said to be periodic, if and only if $f(t+T)=f(t)$ for some value of $T$ and for every value of $t$.

  The smallest positive value of $T$ for which this equation is true for every value of $t$ is called the period of the function.

- If $f(t)$ is a periodic function then

  $$L\{f(t)\} = \frac{1}{1-e^{-sT}} \int_{0}^{T} e^{-st} f(t) dt$$
Lecture-9

INVERSE LAPLACE TRANSFORM

- So far we have considered laplace transforms of some functions $f(t)$. Let us now consider the converse namely, given $F(s)$, $f(t)$ is to be determined. If $F(s)$ is the laplace transform of $f(t)$ then $f(t)$ is called the inverse laplace transform of $f(s)$ and is denoted by $f(t) = L^{-1}\{F(s)\}$
Lecture-10

CONVOLUTION THEOREM

• Let f(t) and g(t) be two functions defined for positive numbers t. We define
  \[ f(t) * g(t) = \int f(u)g(t - u)du \]

• Assuming that the integral on the right hand side exists. f(t)*g(t) is called the convolution product of f(t) and g(t).

• Example: By convolution theorem the value of
  \[ L^{-1}\left\{ \frac{1}{(s-1)(s+2)} \right\} = \frac{e^t - e^{-2t}}{3} \]
Lecture-11

APPLICATION TO DIFFERENTIAL EQUATION

- Ordinary linear differential equations with constant coefficients can be easily solved by the Laplace transform method, without the necessity of first finding the general solution and then evaluating the arbitrary constants. This method, in general, shorter than our earlier methods and is especially suitable to obtain the solution of linear non-homogeneous ordinary differential equations with constant coefficients.
Lecture-12
SOLUTION OF A DIFFERENTIAL EQUATION BY LAPLACE TRANSFORM

• Step 1: Take the laplace transform of both sides of the given differential equation.

• Step 2: Use the formula of Laplace of derivatives
  \[ L\{y^n(t)\} = s^n y(s) - s^{n-1} y'(0) - s^{n-2} y''(0) - \ldots \ldots \ldots - y^{n-1}(0) \]

• Step 3: Replace \(y(0), y'(0)\) etc., with the given initial conditions

• Step 4: Transpose the terms with minus signs to the right

• Step 5: Divide by the coefficient of \(y\), getting \(y\) as a known function of \(s\).

• Step 6: Resolve this function of \(s\) into partial fractions.

• Step 7: Take the inverse laplace transform of \(y\) obtained in step 5. This gives the required solution.
UNIT-V
VECTOR CALCULUS
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In this chapter, vector differential calculus is considered, which extends the basic concepts of differential calculus, such as, continuity and differentiability to vector functions in a simple and natural way. Also, the new concepts of gradient, divergence and curl are introduced.

Example : $i, j, k$ are unit vectors.
VECTOR DIFFERENTIAL OPERATOR

- The vector differential operator is defined as

\[ \nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \]

This operator possesses properties analogous to those of ordinary vectors as well as differentation operator.

Now we will define some quantities known as gradient, divergence and curl involving this operator.
Let $f(x,y,z)$ be a scalar point function of position defined in some region of space. Then gradient of $f$ is denoted by $\nabla f$ or $\nabla f$ and is defined as

$$\text{grad } f = \nabla f = i \frac{\partial f}{\partial x} + j \frac{\partial f}{\partial y} + k \frac{\partial f}{\partial z}$$

Example: If $f=2x+3y+5z$ then grad $f= 2i+3j+5k$
Lecture-3
DIRECTIONAL DERIVATIVE

- The directional derivative of a scalar point function $f$ at a point $P(x,y,z)$ in the direction of $g$ at $P$ and is defined as $\frac{\text{grad } g}{|\text{grad } g|} \cdot \text{grad } f$

- Example: The directional derivative of $f=xy+yz+zx$ in the direction of the vector $i+2j+2k$ at the point $(1,2,0)$ is $10/3$
Lecture-4

DIVERGENCE OF A VECTOR

- Let $f$ be any continuously differentiable vector point function. Then divergence of $f$ and is written as $\text{div } f$ and is defined as

$$\text{div } f = \nabla \cdot f = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

- **Example 1:** The divergence of a vector $2xi + 3yj + 5zk$ is 10

- **Example 2:** The divergence of a vector $f=xy^2 i + 2x^2 yz j - 3yz^2 k$ at $(1,-1,1)$ is 9
SOLENOIDAL VECTOR

- A vector point function $f$ is said to be a solenoidal vector if its divergent is equal to zero i.e., $\text{div } f = 0$
- **Example 1:** The vector $f = (x+3y)i + (y-2z)j + (x-2z)k$ is a solenoidal vector.
- **Example 2:** The vector $f = 3y^4z^2i + z^3x^2j - 3x^2y^2k$ is a solenoidal vector.
Lecture-5
CURL OF A VECTOR

- Let \( f \) be any continuously differentiable vector point function. Then the vector function curl of \( f \) is denoted by \( \text{curl } f \) and is defined as

\[
\text{curl } f = \nabla \times f = i \times \frac{\partial f_1}{\partial x} + j \times \frac{\partial f_2}{\partial y} + k \times \frac{\partial f_3}{\partial z}
\]

- Example 1: If \( f = xy^2 i + 2x^2 yz j - 3yz^2 k \) then \( \text{curl } f \) at \((1,-1,1)\) is \(-i - 2k\)

- Example 2: If \( r = xi + yj + zk \) then \( \text{curl } r \) is 0
IRROTATIONAL VECTOR

- Any motion in which curl of the velocity vector is a null vector i.e., curl $v=0$ is said to be irrotational. If $f$ is irrotational, there will always exist a scalar function $f(x,y,z)$ such that $f=\nabla g$. This $g$ is called scalar potential of $f$.

*Example: The vector $f=(2x+3y+2z)i+(3x+2y+3z)j+(2x+3y+3z)k$ is irrotational vector.*
Lecture-6

VECTOR INTEGRATION

- INTRODUCTION: In this chapter we shall define line, surface and volume integrals which occur frequently in connection with physical and engineering problems. The concept of a line integral is a natural generalization of the concept of a definite integral of $f(x)$ exists for all $x$ in the interval $[a,b]$.
Lecture-7

WORK DONE BY A FORCE

- If $F$ represents the force vector acting on a particle moving along an arc $AB$, then the work done during a small displacement $F.dr$. Hence the total work done by $F$ during displacement from $A$ to $B$ is given by the line integral $\int F.dr$

- Example: If $f=(3x^2 + 6y)i - 14yzj + 20xz^2 k$ along the lines from $(0,0,0)$ to $(1,0,0)$ then to $(1,1,0)$ and then to $(1,1,1)$ is $23/3$
Lecture-8
SURFACE INTEGRALS

• The surface integral of a vector point function $F$ expresses the normal flux through a surface. If $F$ represents the velocity vector of a fluid then the surface integral $\int F \cdot n \, ds$ over a closed surface $S$ represents the rate of flow of fluid through the surface.

• **Example:** The value of $\int F \cdot n \, ds$ where $F = 18zi - 12j + 3yk$ and $S$ is the part of the surface of the plane $2x + 3y + 6z = 12$ located in the first octant is 24.
Lecture-9

VOLUME INTEGRAL

Let \( f(r) = f_1 \mathbf{i} + f_2 \mathbf{j} + f_3 \mathbf{k} \) where \( f_1, f_2, f_3 \) are functions of \( x, y, z \). We know that \( dv = dx dy dz \).

The volume integral is given by

\[
\iiint f dv = \iiint (f_1 \mathbf{i} + f_2 \mathbf{j} + f_3 \mathbf{k}) dx dy dz
\]

Example: If \( F = 2xz \mathbf{i} - xj + y^2 \mathbf{k} \) then the value of \( \int \int \int f dv \) where \( v \) is the region bounded by the surfaces \( x = 0, x = 2, y = 0, y = 6, z = x^2, z = 4 \) is \( 128 \mathbf{i} - 24 \mathbf{j} - 384 \mathbf{k} \)
VECTOR INTEGRAL THEOREMS

- In this chapter we discuss three important vector integral theorems.
  1) Gauss divergence theorem
  2) Green’s theorem
  3) Stokes theorem
Lecture-10

GAUSS DIVERGENCE THEOREM

This theorem is the transformation between surface integral and volume integral. Let S be a closed surface enclosing a volume v. If \( f \) is a continuously differentiable vector point function, then

\[
\int \text{div} \, f \, dv = \int f \cdot nds
\]

Where \( n \) is the outward drawn normal vector at any point of \( S \).
Lecture-11
GREEN’S THEOREM

This theorem is transformation between line integral and double integral. If S is a closed region in xy plane bounded by a simple closed curve C and in M and N are continuous functions of x and y having continuous derivatives in R, then

\[ \int Mdx + Ndy = \iint \left[ \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right] dxdy \]
STOKES THEOREM

- This theorem is the transformation between line integral and surface integral. Let S be an open surface bounded by a closed, non-intersecting curve C. If F is any differentiable vector point function then

\[ \int F \cdot dr = \int \text{Curl} f \cdot nds \]